Reg. No. | | | | | | | | | | |

ANNA UNIVERSITY (UNIVERSITY DEPARTMENTS)

B.E./ B.Tech (Full Time) - END SEMESTER EXAMINATION, Nov / Dec 2024

INFORMATION TECHNOLOGY

MA5302 - DISCRETE MATHEMATICS

(Regulation: 2019)

Time: 3 hours **Maximum Marks: 100**

CO1	To introduce Mathematical Logic, Inference Theory and proof methods.
CO2	To provide fundamental principles of combinatorial counting techniques.
CO3	To introduce graph models, their representation, connectivity and traversability.
CO4	To explain the fundamental algebraic structures, groups and their algebraic properties.
CO5	To provide exposure to the development of the algebraic structures, lattices and Boolean algebra and to demonstrate the utility of Boolean laws.

BL – Bloom's Taxonomy Levels
(L1-Remembering, L2-Understanding, L3-Applying, L4-Analysing, L5-Evaluating, L6-Creating)

PART- A (10x2=20Marks)
(Answer all Questions)

Q. No.	Questions	Marks	CO	BL
1	Find the truth table of $P \to (Q \to \neg P)$.	**2**	1	1
2	Let $A = \{1, 2, 3, 4, 5, 6\}$. Determine the truth value of each of the following: i) $(\exists x \in A)\,(x^2 > 25)$, ii) $(\forall x \in A)\,(x^2 - x \le 30)$.	**2**	1	1
3	How many different permutations are there in the word ENGINEERING?	**2**	2	2
4	A license plate consists of two letters of the English alphabet followed by four digits. How many license plates are there?	**2**	2	2
5	Is it possible to have a graph with 7 vertices each of degree 3? Justify your answer.	**2**	3	1
6	For what values of n the complete graph with n vertices is Eulerian.	**2**	3	2
7	Show that every subgroup of a cyclic group is a normal subgroup.	**2**	4	1
8	Give an example for a ring.	**2**	4	1
9	Define partial ordered relation.	**2**	5	1
10	Give an example of a lattice which is complemented but not distributive.	**2**	5	1

PART- B(5x13 = 65 Marks)
(Answer all Questions)

Q. No.		Questions	Marks	CO	BL
11 (a)	(i)	Obtain the principal conjunctive normal form of. $(P \to R) \wedge (P \to Q) \wedge (\neg Q \to P)$ by using equivalences.	5	1	3
	(ii)	Show that $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$ by using proof by contradiction.	8	1	3
		OR			
11 (b)	(i)	Show that $((P \vee Q) \wedge 7(7P \wedge (7Q \vee 7R))) \vee (7P \wedge 7Q) \vee (7P \wedge 7R)$ is a tautology by without using truth table.	5	1	3
	(ii)	Show that $R \to S$ is logically derived from the premises $P \to (Q \to S)$, $\neg R \vee P$ and Q.	8	1	3
12 (a)	(i)	Find the number of integers between 1 and 1000 that are divisible by 3 or 5.	5	2	4
	(ii)	Solve the recurrence relation $a_n - 6a_{n-1} + 9a_{n-2} = 2n+1$, for $n \ge 2$ with initial conditions $a_0 = 0$ and $a_1 = 1$.	8	2	4
		OR			
12 (b)	(i)	Find the generating function to recurrence relation $S(n+1) - 2S(n) = 4^n$ with $S(0) = 1,\ n \ge 0$.	5	2	4
	(ii)	Using mathematical induction show that $\sum_{r=0}^{n} 3^r = \frac{3^{n+1}-1}{2}$.	8	2	4
13 (a)		If G is a connected simple graph with n vertices with $n \ge 3$, such that the degree of every vertex in G is at least $\frac{n}{2}$, then prove that G is Hamiltonian graph. Is that condition a necessary for Hamiltonian graph? Justify your answer.	13	3	3
		OR			
13 (b)	(i)	Prove that the number of odd degree vertices in any graph is even.	5	3	3
	(ii)	For what values of n, a graph with n vertices can be self complementary? Justify your answer. By applying your answer, find the values of n so that the path P_n is self complementary.	8	3	3

Q. No.		Questions	Marks	CO	BL
14 (a)		State and prove Lagrange's theorem on groups.	13	4	4
		OR			
14 (b)	(i)	Prove that every subgroup of a cyclic group is cyclic.	5	4	4
	(ii)	Let $f : G \rightarrow H$ be a homomorphism from the group $\langle G, * \rangle$ to the group $\langle H, \Delta \rangle$. Prove that the kernel of f is a normal subgroup of G.	8	4	4
15 (a)	(i)	Show that every chain is modular.	5	5	3
	(ii)	Show that following in a complemented distributive lattice. $a \leq b \Leftrightarrow a \wedge b' = 0 \Leftrightarrow a' \vee b = 1 \Leftrightarrow b' \leq a'$. Where a' denotes the complement of a.	8	5	3
		OR			
15 (b)		Show that every chain is a distributive lattice. Justify whether converse of the above statement is true or not.	13	5	3

PART- C (1x 15=15Marks)
(Q.No.16 is compulsory)

Q. No.		Questions	Marks	CO	BL
16.	(i)	Draw the graphs represented by the following adjacency matrix and exhibit an isomorphism or provide a rigorous argument that none exists. $\begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$	7	3	5
	(ii)	Show that "It rained" is a conclusion obtained from the following statements. "If it does not rain or if there is no traffic dislocation, then the sports day will be held and the cultural programme will go on". "If the sports day is held, the trophy will be awarded" and "the trophy was not awarded".	8	1	6

*** END ***